On the Statistical Precision of the Economic and Fiscal Support Ratios

Iván Mejía-Guevara

December 2009

Abstract

Important estimates are usually obtained in socioeconomic studies through the use of survey data. Consequently, the statistical validation of those estimates is a matter of important concern for diverse reasons, like differences on the quality of information, for instance. This article focuses on the definition of basic but important statistical properties in the estimation of the support ratio, a concept that is defined in the context of the National Transfer Accounts (NTA) project by means of the lifecycle of production and consumption. A fiscal support ratio is also defined in a similar way. The survey design is taken into account for the variance estimation and evidence from Mexico is reported for three years. Some relevant details that emerge in practice are discussed too.

1 Introduction

The interaction between the economic lifecycle and the age distribution is a central element in the study of intergenerational reallocations. The economic lifecycle is defined in terms of the lifecycle of production and consumption, which can be estimated in practice following the National Transfer Accounts framework (Mason et al., 2009). Other important relationships can be obtained from the age profiles of consumption and production like the economic support ratio (ESR). The economic support ratio is defined by Mason and Lee (2006) as an alternative measure of dependency that explicitly incorporates age-variation in consumption and labor productivity. The values at each age are used as population weights to provide estimates of the effective number of consumers and the effective number of producers. Then, if $L(t)$ and $N(t)$ represent the effective number of producers and consumers, respectively; the support ratio is the effective number of producers per consumer:

$$SR = \frac{L(t)}{N(t)} = \frac{\sum_a^A \gamma(a)P(a,t)}{\sum_a^A \phi(a)P(a,t)} \tag{1}$$

where $P(a,t)$ is the population aged at time t, A is the age of the oldest individual in the population and $\phi(a)$, $\gamma(a)$ are age-specific, time-invariant that measure age variation of consumption and productivity, respectively. A fiscal support ratio is defined in similar way if $\phi(a)$, $\gamma(a)$ represent now time-invariant measures of public transfers and taxes, respectively. In this case, what is obtained is an alternative measure of public finance dependency that incorporates age-variation in public transfers and taxes. The age especific profiles for taxes, social benefits and transfers can also be estimated via the NTA approach (Mason et al., 2009).

In practice, both the ERS and the FSR have important applications. For instance, the ESR is formalized in (Mason and Lee, 2006) and used in the definition of the first demographic dividend. Then, the demographic dividend (defined as the chage in the support ratio) represents the boost to the growth rate of per capita income which arises when the proportion of people in the working ages rises relative to the sum of the young and the elderly.

However, although some evidence has been reported about these important concepts (Mason and Lee 2006)(Mason 2007), no statistical properties have been reported for these estimates. Therefore, I focus in this article on the determination and survey design considerations for basic statistical properties for the ESR and the FSR within the NTA framework. In particular, I propose the determination of standard errors and confidence intervals by considering the sampling design of the survey (or surveys), since this is the main source of information used to compute these assessments. The consideration of the statistical survey design is important because the variance estimation might differ significantly due to this fact. In particular, we propose here the case when a multi-stage cluster sampling design is employed, given that this kind of design generally leads to estimates with bigger variance as opposed to the variance of estimates obtained trough a Simple Random Sample (SRS) design (Kish, 1965) (Heeringa et al., forthcoming). Despite of the variance is a classical and simple measure of variability, it is a very important concept since survey data are the predominant source of information for socioeconomic studies.

2 The variance estimation of support ratios: a complex survey design approach

The main purpose of this article relies on the fact that the variance of the SR can be estimated as the variance of an statistical ratio in the context of a complex design survey -an stratified multi-stage cluster sample design. This is clear when we observe the definition of a ratio as a survey estimate, given by the following expression:

$$\hat{r} = \frac{x}{z} = \frac{\sum_i^n w_i x_i}{\sum_i^n w_i z_i} \tag{2}$$

where x_i and z_i the variables of interest for the individual i in the sampling population, n the sample size and w_i is the sample weight of individual i, which reflects the number of people she represents in the actual population (inflation factor). The variance of the expression (2) can not be estimated directly, since both terms x_i, y_i are random variables and $Var\left(\frac{x}{z}\right) \neq \frac{Var(x)}{Var(z)}$. In this case, a different approach is necessary to compute it. Some approaches generally used in practice are: the Taylor Series Linearization (TSL) method or replication methods for variance estimation (Heeringa et al., forthcoming) -like Jacknife Repeated Replication (JRR), Balanced Repeated Replication (BRR) and the Bootstrap. For its simplicity and due to the characteristics of the data used here, I employ the TSA (the default method for most contemporary software package that compute sampling variances for complex sample survey data). Then, the variance estimation of the expression (2), when using the TSA method, is as follows:

$$var(\hat{r}) = \frac{1}{x^2}[var(x) + \hat{r}^2 \cdot var(z) - 2 \cdot \hat{r} \cdot cov(x, z)] \tag{3}$$

Returning to the definition of the SR, if we observe the expressions (1) and (2), we can appreciate their similarity; however, we could not apply equation (3) directly for the variance estimation of (1) since the sampling weights and the complex sampling design need to be considered. However, this can be easily fixed if the expression (1) is rearranged. First, let us recall that $\gamma(a)$ and $\phi(a)$ are sampling estimates, which represent per capita values for an individual of age a. Then, they are computed by

$\hat{\gamma}(a) = \frac{\sum_j^{n(a)} w(a)_j \gamma(a)_j}{\sum_j^{n(a)} w(a)_j}$, $\hat{\phi}(a) = \frac{\sum_j^{n(a)} w(a)_j \phi(a)_j}{\sum_j^{n} w(a)_j}$ where $\sum_j^{n(a)} w(a)_j = W(a)$, $W(a)$ being the size of the weighted population of age a and $n(a)$ the number of individual aged a in the the survey. The next step consists of replacing the survey estimates $\hat{\gamma}(a)$ and $\hat{\phi}(a)$ into the expression (1). However, some underlying and important aspects in the determination of equation (1) must be taken into considration since they might alter the proper estimation of the variance: a) the case when the size of the weighted population in the survey does not match with the actual population (the one which is obtained from an official source, like government's reports), that is $W(a) \neq P(a,t)$, b) a macroeconomic adjustment (MA) that is applied to age profiles in the NTA methodology to adjust the difference between what is reported in the survey and National Accounts (Lee et al., 2008); that is, the MA for the survey estimates $\hat{\gamma}(a)$, $\hat{\phi}(a)$ when $\hat{\gamma}(a) \neq \gamma(a)$ and $\hat{\phi}(a) \neq \phi(a)$. Three scenarios are considered here in order to deal with these adjustments:

(i) $W(a) = P(a,t)$ and no MA

Let us consider for the moment just the numerator in expression (1), then the replacement of $\gamma(a)$ by $\hat{\gamma}(a)$, leads us to:

$$\hat{L}(t) = \sum_a^A \hat{\gamma(a)} P(a,t) = \sum_a^A \frac{\sum_j^{n(a)} w(a)_j \gamma(a)_j}{\sum_j^{n(a)} w(a)_j} P(a,t) \tag{4}$$

but given that $\sum_j^{n(a)} w(a)_j = W(a) = P(a,t)$, we have $\hat{L}(t) = \sum_a^A \sum_j^{n(a)} w(a)_j \gamma(a)_j$, which can be re-expressed in an equivalent way as $\hat{L}(t) = \sum_k^n w_k \gamma_k$. Following the same procedure, we get $\hat{N}(t) = \sum_k^n w_k \phi_k$, where γ_k and ϕ_k are the sample values of γ and ϕ for the individual k. In this case, we apply the equation (3) directly to $\hat{L}(t)/\hat{N}(t)$ get the variance.

(ii) $W(a) \neq P(a,t)$ and no MA

If the total population does not match with the official source, then we have:

$$\hat{L}(t) = \sum_a^A \hat{\gamma(a)} P(a,t) = \sum_a^A \left[\frac{\sum_j^{n(a)} w''(a)_j \gamma(a)_j}{\sum_j^{n(a)} w''(a)_j} \right] P(a,t) \tag{5}$$

where $P(a,t) = \sum_j^{n(a)} w''(a)_j$ and $w''(a)_j = w'(a)_j * w(a)_j$, being $w(a)_j$ the original sample weights, and $w'(a)_j$ is a new weight which is necessary to apply to individual j in the age group a to match the totals of survey and actual populations. This new weight can be obtained as $W'(a)/W(a)$; that is, $W'(a)$ is the total population of age a in the official source and $W(a)$ is the weighted popualtion of age a in the sample $W(a) = \left[\sum_j^{n(a)} w(a)_j\right]$. In other words, $w''(a)_j = W'(a)/W(a) * w(a)_j$ and, therefore, the expression in (5) becomes:

$$\hat{L}(t) = \sum_a^A \left(\frac{W'(a)}{\sum_j^{n(a)} w(a)_j} \right) \left[\sum_j^{n(a)} w(a)_j \gamma(a)_j \right] \tag{6}$$

Similarly,

$$\hat{N}(t) = \sum_a^A \left(\frac{W'(a)}{\sum_j^{n(a)} w(a)_j} \right) \left[\sum_j^{n(a)} w(a)_j \phi(a)_j \right] \tag{7}$$

Yet, from (6) and (7), we have the problem that $\sum_j^{n(a)} w(a)_j$ is a random variable and it difficults the application of the TSL method directly, since both γ and ϕ have this term in the denominator.

One alternative I propose to deal with this drawback consists on estimating the weights $w^{''}$ first and to use them instead of the original ones, as in the case (i), to get:

$$\hat{SR} = \frac{\hat{L(t)}}{\hat{N(t)}} = \frac{\sum_k^n w^{''}(a)_k \gamma(a)_k}{\sum_k^n w^{''}(a)_k \phi(a)_k} \tag{8}$$

then applied the TSL procedure to this term directly.

(iii) $W(a) = P(a,t)$ and MA

The macroeconomic adjustment for the components of the SR can be expressed as: $\gamma(a)_j = \beta^{\gamma} * \hat{\gamma}(a)_j$, $\phi(a)_j = \beta^{\phi} * \hat{\phi}(a)_j$, where β^{γ} and β^{ϕ} are the adjustment factors or the fraction between survey and the national values, for γ and ϕ, respectively.

Since $W(a) = P(a,t)$, the only adjustment in equation (1) is the substitution of the survey estimtes $\hat{\gamma}$ and $\hat{\phi}$ after the macroadjustment. Then, (1) becomes:

$$\hat{SR} = \frac{\sum w_k \beta^{\gamma} \gamma(a)_k}{\sum w_k \beta^{\phi} \hat{\phi}(a)_k} = \frac{\sum w_k^{\beta^{\gamma}} \gamma_k}{\sum w_k^{\beta^{\phi}} \phi_k} \tag{9}$$

Therefore, the variance of (9) can be computed as in the case (i). Moreover, another alternative may be used since β^{γ} and β^{ϕ} are constants, which simplifies the variance approximation using the equation: $var(\hat{SR}) = \left(\frac{\beta^{\gamma}}{\beta^{\phi}}\right)^2 * var\left(\hat{SR}_{(i)}\right)$ where $var(\hat{SR}_{(i)})$ stands for the varince of the $\hat{SR}$ obtained following the case (i). From this last expression we can notice that, apart from the possible increase in variance due to $W(a) \neq P(a)$, the macroeconomic adjustment implies an increase in variance only if the adjustment in labor income surpasses the adjustment of total consumption; otherwise, we obtain a gain in precision if the opposite occurs. In practice, the common case in the NTA methodology is $W(a) \neq P(a)$ and a macroeconomic adjustment is necessary. In this case, combining the options (ii) and (iii) leads us to the proper estimation of the variance -adjusting the population first and then applying case (iii). I have exposed cases (i) to (iii) to illustrade possible differences in precission and implications among them. In what follows, I will present evidence from Mexico for three years, which will also provide a way for longitudinal comparisons for the precision of the estimates.

3 Data Processing and Other Considerations

3.1 The data

The main source of information for estimating the support ratio in Mexico is the national survey of income and expense for homes (called ENIGH) which is conducted by the National Statistical Institute (INEGI). Although the survey has no established periodicity, since 1992 there has been one every two years (there is also one for 2005, a special case obtained given the *Conteo 2005*, a lower-scale census also conducted decennially, the timing of which is offset from the General Census by five years). The ENIGH structure is cross-sectional which covers households composed of nationals and foreigners (excluding diplomats), who usually reside in private dwellings (excluding institutional or collective dwellings) in the whole territory. The geographic information is at Federal district level. Three years are reported in this paper where the sampling sizes are reported in the next table:

year	hh units	hh members
2000	10,108	42,535
2002	17,167	72,602
2004	22,595	91,738

The ENIGH employs an stratified multi-stage cluster sample design. The stratification was made according to 4 geographic regions which are determined according to the size of the locality (a political subdivision within municipalities). Codes for stratification for the 4 strata are reported in the public dataset, as well as for stratification by socioeconomic level. Those strata are mixed to define combined strata. Although no explicit variable is reported for the Primary Sampling Units (PSUs), a geographic variable which specifies codes for states and municipalities is used as PSU. These variables are devoted to build a sampling error computation model, where I use the random groups method to combine multiple clusters from each single design stratum. I named the variable containing the sampling error cluster codes as sampling error computation unit (SECU). This was implemented to facilitate the variance estimation (Heeringa et al. 2010). At the end 2 SECUs were obtained for each stratum which amounts for 16 SECUs.

For macroeconomic adjustment, information is taken from the Ministry of Finance (SHCP) and INEGI. As stated above, I provide evidence for three years: 2000, 2002 and 2004. This years were chosen since complete information is available for them in Mexico, through the use of the three public surveys: ENIGH-2000, ENIGH-2002 and ENIGH-2004. Life-cycle estimates have been already reported in (Mejía-Guevara et al., forthcoming), taken the age profiles for the life-cycle reported in (Mejía-Guevara, 2008, forthcoming), but **no** statistical precission has been reported for those estimates until now.

3.2 Economic support ratio

Some important aspects in the construction of the ESR must be taken into consideration. First of all, both labor income and consumption profiles are constructed as the sum of different elements according to NTA methods (Lee etal., 2008). In the case of Mexico, labor income includes earnings and unincorporated income, whereas consumption is made of public and private consumption, both including education, health and other consumption. Private consumption also incorporates the imputing rent from the own house or housing (Mejía-Guevara, 2008). Different methods are applied in the allocation by age for these profiles, for example, the age profile for private education is assembled by using a regression approach (Mejía-Guevara, 2008). See (Lee etal., 2008) for more information regarding the allocation by age of the NTA's profiles. This is relevant since the approach handled here is assuming that the possible variability for each element is captured in the construction of the variance using the TSL method given by the formula (3).

Moreover, another important subject is that only the allocation by age of the private consumption is computed through the use of survey data, whereas for the public profiles only the education profile uses enrollment rates obtained from survey data in its creation. Health and other public consumption is obtained from other sources (Mejía-Guevara, 2008). The inclusion of public consumption deserves an additional consideration in terms of the variance estimation since the total consumption involves private and public consumption. In order to deal with this issue, I follow a simple approach which consists on adding the final public consumption profile to the sample database. The term "adding" refers to including additional weights into the sample in order to account for the public consumption there. However, this has an effect on the expression (9) that has to be encompassed. To better appreciate this, let's make some little changes in the notation being $yl = \gamma$ and $c = \phi$. Thenceforth, two simple approaches may be followed to fix this drawback: (a) modifying the denominador in expression (9) as follows: $\frac{\sum w_k \beta^{yl} yl_k}{\sum w_k \left[\beta^f c_k^f + c_k^g\right]}$, being c_k^f, c_k^g private and public consumption profiles, respectively or, (b) using the expression: $\frac{\sum w_k \beta^y yl_k}{\sum w_k \beta^c c_k}$, where β^c is the factor that results as a consecuence of adding up c_k^p and c_k^g after the macroeconomic adjustment, performed like in case (iii). Either (a) and (b) have the drawback that the variance will reflect a factor -the public consumption- which was not constructed "directly" from the survey, but it has to be added given the definition of the economic support ratio. Although, option (a) is prefered since the proper weight is applied to private consumption. On the other side, since we are implicitly considering the age distribution, adjusted via the survey information, we can assume that the variability of the survey given by the cluster design might reflect part of the true -unknown- variability of the public consumption. In the next sub-section I perform this two approaches and a third one in which no public consumption is contemplated to get some evidence about the impact of public consumption on the overall precission of the ESR.

References

Heeringa, Steve, Brady West, and Patricia Berglund (forthcoming). Applied Survey Data Analysis. Chapman and Hall, Publication date: 03/12/2010.

Kish, Leslie (1965). Survey Sampling. John Wiley and Sons, Inc.

Lee, Ronald, Sang-Hyop Lee, and Andrew Mason (2008). Charting the economic lifecycle. In A. Prskawetz, D. E. Bloom, and W. Lutz, (eds.), Population aging, human capital accumulation, and productivity growth. Population and Development Review 33(Suppl.). New York: Population Council. Also NBER Working Paper No. 12379.

Mason, Andrew and Ronald Lee (2006). "Reform and Support Systems for the Elderly in Developing Countries: Capturing the Second Demographic Dividend," GENUS LXII (2) 11-35.

Mason, Andrew, Ronald Lee, An-Chi Tung, Mun-Sim Lai, and Tim Miller (2009). Population aging and intergenerational transfers: Introducing age into national accounts. In D. Wise (ed.), Developments in the Economics of Aging. Chicago: National Bureau of Economic Research and University of Chicago Press (forthcoming).

Mason, Andrew (2007). Demographic Transition and Demographic Dividends in Developing and Developed Countries, Proceedings of the United Nations Expert Group Meeting on Social and Economic Implications of Changing Population Age Structures, Mexico City, 31 August 2 September 2005.

Mejía-Guevara, Iván (2008). Ciclo de Vida Económico en México (Economic Life Cycle in Mexico). En F. Vélez Fernández-Varela (coord.), La Situación Demográfica de México 2008. México D. F.: Consejo Nacional de Población.

Mejía-Guevara, Iván (forthcoming). Economic Life Cycle and Intergenerational Redistribution: Mexico, 2004. Poster presented at the XXVI IUSSP International Population Conference, Marrakech, Morocco.

Mejía-Guevara, Iván, Félix Vélez Fernández-Varela, and Juan E. García-López (forthcoming). El Primer Dividendo Demográfico y los Esquemas de Protección Social en México (The First Demographic Dividend and the Social Protection Schemes in Mexico). Presented at the Latin American NTA Seminar on Population Aging, Intergenerational Transfers, and Social Protection, Santiago, Chile, Oct. 2009.